ABELIAN BY SIMPLE FINITE MOUFANG LOOPS

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Recall that a loop M is *Moufang* if the identity $xy \cdot zx = (x \cdot yz)x$ holds for all $x, y, z \in M$. Suppose there is a short exact sequence of finite Moufang loops

$$1 \to U \to E \to M \to 1,$$

where U is an abelian group. We will identify U with its image in E and say that the extension E is *minimal*, if U contains no subgroup that is a normal subloop of E. The case where M is simple is of special interest due to the classification [1].

We give explicitly two construction of minimal extensions of abelian groups by simple Moufang loops. The first one is based on the correspondence between Moufang loops and groups with triality. Given a group G, any RG-module V gives rise to a module for the *wreathlike* triality group $G \times G \times G$ on which S_3 acts by permuting the three factors. We obtain a criterion when this module admits triality and write a multiplication formula in the corresponding Moufang loop which we call a *Moufang semidirect product* $G \leftthreetimes V$.

The second construction may be viewed as a generalization of group action on associative algebras to Moufang loop 'action' on alternative algebras. Whenever a Moufang loop M is mapped to invertible elements $A^{\times}$ of an alternative algebra A, one can always construct an *outer semidirect product* $M \leftthreetimes U$ which is a Moufang loop, where U is any factor group of the additive group of A invariant under the operators $L_{m,n}$ and T_m for $m, n \in M$.

Thus the known nontrivial (i. e. nonassociative and not of the form $U \times M$) minimal extensions of finite simple noncyclic Moufang loops are as follows:

- Moufang semidirect products $G \leftthreetimes V$ for a finite simple group G.
- Outer semidirect products $M \leftthreetimes U$ for a finite simple Paige–Moufang loop M and an irreducible factor space U of the corresponding Cayley algebra.
- Nonsplit central extensions $1 \to \mathbb{Z}_2 \to E \to M(q) \to 1$, where q is an odd prime power or $q = 2$, and $M(q)$ is the the simple Paige–Moufang loop over $\mathbb{F}_q$.

The extensions in the last case are isomorphic to the elements of norm 1 of the finite Cayley algebra $\mathbb{O}(q)$ if q is odd, and to the exceptional double cover of $M(2)$ of order 240 if $q = 2$. We put forward

Conjecture. *Up to isomorphism, the only nontrivial minimal extensions for finite simple noncyclic Moufang loops are those given in the list above.*

We also remark that the case where M is cyclic was treated in [2].

References

1. Liebeck M. W. The classification of finite simple Moufang loops // Math. Proc. Camb. Phil. Soc. 1987. V. 102. No. 1. P. 33–47.

2. Grishkov A. N., Zavarnitsine A. V. Abelian-by-cyclic Moufang loops // Comm. Alg. 2013. V. 41. No. 6. P. 2242–2253.